Rubin Causal Model

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Notes

Outline

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Notes

Potential outcomes framework

There are two possible outcomes: $Y_i(1)$ if individual i undergoes treatment T and $Y_i(0)$ if he does not. *These are not random.*

T_i is an indicator of treatment status. The treatment effect for i is $\tau_i = Y_i(1) - Y_i(0)$. Hence,

$$Y_i = (1 - T_i)\, Y_i(0) + T_i\, Y_i(1) = Y_i(0) + T_i \tau_i.$$

τ_i varies across individuals, giving it a distribution in the population, even though it is fixed for an individual.

Treatment status is a random variable.

Notes

Fundamental problem of causal inference

Why not just estimate τ_i directly?
We only observe *either* $Y_i(0)$ *or* $Y_i(1)$, not both. This is the *fundamental problem of causal inference.*

At its core, causal inference is a *missing data problem.*

Notes

SUTVA

We have already made an important assumption: observation i's outcome only depends upon his treatment status—not anyone else's. This rules out:

- General equilibrium effects (doubling one person's income while keeping everyone else's the same versus doubling everyone's income)
- Interaction effects/network effects/spill-overs/externalities (not vaccinating one person when everyone else is vaccinated versus the opposite)

This is known as the *stable unit treatment value assumption* (SUTVA).

Notes

Treatment effects

The two main effects of interest are the *average treatment effect (ATE)* and the *average treatment effect for the treated (ATT):*

$$
\begin{aligned}
\text{ATE} &= \mathbb{E}(\tau_i) \\
\text{ATT} &= \mathbb{E}(\tau_i | \underbrace{T_i = 1}_{\text{Distribution of treated}}) \\
&= \mathbb{E}_1(\tau_i)
\end{aligned}
$$

Why are these different? Selection into treatment. These effects are often conditioned on a set of predictors X:

$$
\begin{aligned}
\text{ATE}(x) &= \mathbb{E}(\tau_i | X = x) \\
\text{ATT}(x) &= \mathbb{E}_1(\tau_i | X = x)
\end{aligned}
$$

Notes

Under treatment independence

Suppose that treatment is *randomly assigned:*

$$(Y_i(0), Y_i(1)) \perp T_i.$$

Then:

$$\begin{aligned}\mathbb{E}_1[\tau_i] &= \int \tau f(\tau \mid T = 1)\, d\tau \\ &= \int \tau f(\tau)\, d\tau \\ &= \mathbb{E}[\tau_i];\end{aligned}$$

the ATT = ATE.

Notes

Conditional expectations

Similarly, we can show that:

$$\begin{aligned}\mathbb{E}_0(Y) &= \mathbb{E}_0\left[(1 - T_i)\, Y_i(0) + T_i\, Y_i(1)\right] \\ &= \mathbb{E}_0\left[Y_i(0)\right] = \mathbb{E}\left[Y_i(0)\right]\end{aligned}$$

and:

$$\begin{aligned}\mathbb{E}_1[Y] &= \mathbb{E}_1\left[(1 - T_i)\, Y_i(0) + T_i\, Y_i(1)\right] \\ &= \mathbb{E}_1\left[Y_i(1)\right] = \mathbb{E}\left[Y_i(1)\right].\end{aligned}$$

Hence:

$$\begin{aligned}\text{ATE} = \text{ATT} &= \mathbb{E}\left[Y_i(1) - Y_i(0)\right] = \mathbb{E}\left[Y_i(1)\right] - \mathbb{E}\left[Y_i(0)\right] \\ &= \mathbb{E}_1\left[Y_i(1)\right] - \mathbb{E}_0\left[Y_i(0)\right]\end{aligned}$$

Notes

Overcoming the fundamental problem

$$\text{ATE} = \text{ATT} = \mathbb{E}_1\left[Y_i(1)\right] - \mathbb{E}_0\left[Y_i(0)\right]$$

Random assignment allows us to use the control observations to fill in the missing outcomes for the treated observations (on average).

Notes

Under conditional independence

How can this problem be solved in the absence of random treatment assignment?

We need the assumption of *unconfoundedness* or *selection on observables*:

$$(Y_i(0), Y_i(1)) \perp T_i | X_i.$$

Notes

Confirmation of previous results

We can extend the previous results conditional on X:

$$\text{ATE}(x) = \text{ATT}(x) = \mathbb{E}_1 [Y | X = x] - \mathbb{E}_0 [Y | X = x]$$

Hence, to find $\text{ATT}(x)$, find the average value of Y for those members of the treated group with $X = x$ and subtract it from the average Y for the control population with $X = x$.

We can use several different approaches to estimate this quantity.

Notes

RCM and identification strategies

Randomized studies (also natural experiments and matching):
Rubin, Donald B. 1974. "Estimating Causal Effects of Treatments in Randomized and Nonrandomized Studies." *Journal of Educational Psychology.* 66: 688–701.

Instrumental variables:
Angrist, Joshua D., Guido W. Imbens and Donald B. Rubin. 1996. "Identification of Causal Effects Using Instrumental Variables." *Journal of the American Statistical Association.* 91(434): 444-455.

Regression discontinuity:
Lee, David S. 2008. "Randomized Experiments from Non-random Selection in U.S. House Elections." *Journal of Econometrics.* 142(2): 675–697.

Difference-in-differences:
Abadie, Alberto. 2005. "Semiparametric Difference-in-Differences Estimators." *Review of Economic Studies.* 72: 1–19.

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